Blurred Image Restoration Using the Type of Blur and Blur Parameters Identification on the Neural Network

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ABSTRACT

As a rule, blur is a form of bandwidth reduction of an ideal image owing to the imperfect image formation process. It can be caused by relative motion between the camera and the original scene, or by an optical system that is out of focus.

Today there are different techniques available for solving of the restoration problem including Fourier domain techniques, regularization methods, recursive and iterative filters to name a few. But without knowing at least approximate parameters of the blur, these filters show poor results. If incorrect blur model is chosen then the image will be rather distorted much more than restored.

The original solution of the blur and blur parameters identification problem is presented in this paper. A neural network based on multi-valued neurons is used for the blur and blur parameters identification. It is shown that using simple single-layered neural network it is possible to identify the type of the distorting operator. Four types of blur are considered: defocus, rectangular, motion and Gaussian ones. The parameters of the corresponding operator are identified using a similar neural network.

After a type of blur and its parameters identification the image can be restored using several kinds of methods. Some fundamentals of image restoration are also considered.

Keywords: Image restoration, neural network, frequency domain

1. INTRODUCTION

As a rule, blur is a form of bandwidth reduction of an ideal image owing to the imperfect image formation process. It can be caused by relative motion between the camera and the original scene, or by an optical system that is out of focus. When aerial photographs are produced for remote sensing purposes, blurs are introduced by atmospheric turbulence, aberrations in the optical system, and relative motion between the camera and the ground. Such blurring is not confined to optical images, for example electron micrographs are corrupted by spherical aberrations of the electron lenses, and CT scans suffer from X-ray scatter.

Today there are different techniques available for solving of the restoration problem including Fourier domain techniques, regularization methods, recursive and iterative filters[1,2], to name a few. All of the existing techniques are directed to the obtaining of a solution for the deconvolution problem. But without knowing at least approximate parameters of the blur, these filters show poor results. If incorrect blur model is chosen then the image will be rather distorted much more than restored. All of the known filters are trying to build a model of blur: blurring function and its point spread function. More complex of them also try to make some assumptions about the ideal image and even create some approximation of it. Many of different algorithms for blur identification and identification of its parameters exist today, for example, the maximum likelihood blur estimation[3] or regularization approach[4]. The disadvantage of these algorithms is their computing complexity and relatively high level of the misidentification.

In this paper we would like to present an original solution of this problem. As it was said, to restore the blurred image, it is very important to know the type of the blur and to estimate its parameters. This knowledge is the knowledge of the

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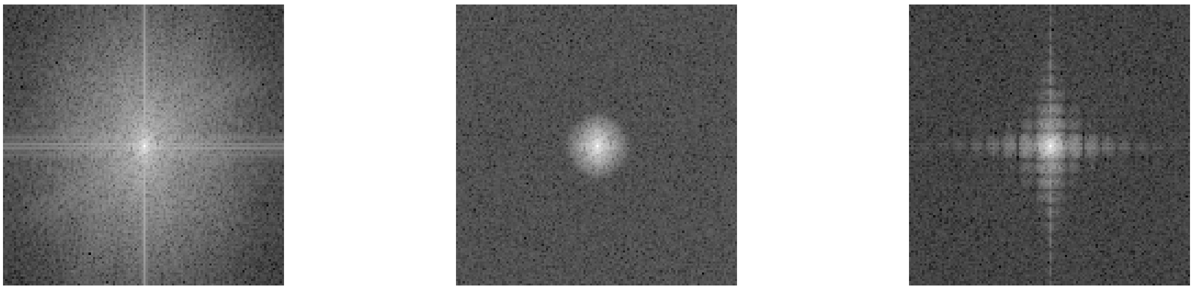
mathematical model of the corresponding distorting operator. A precise estimation of the parameters, which the corresponding model depends on, is also very important for the restoration of the blurred image.

The background for our solution is based on the learning of the specific distortions that are implied by the distorting operator in the Fourier spectrum amplitude (see Fig. 1). To identify the distorting operator, its mathematical model and its parameters, we will put this problem to the field of the pattern recognition. So on our consideration a type of blur identification is a classification problem. The estimation of the blur parameters is also a classification problem.



(a) (b) (c)

Fig. 1. Influence of the blur on Fourier spectrum amplitude: (a) – spectrum amplitude of the image that is not corrupted; (b) – spectrum amplitude of the same image corrupted by the Gaussian blur; (c) – spectrum amplitude of the same image corrupted by the rectangular blur

To solve the classification problem, we will use a neural network based on multi-valued neurons (MVN)[5]. It will be used for the recognition (identification) of the distorting operator or kind of blur. A similar MVN-based neural network will be used to recognize the corresponding distorting operator parameters. The multi-valued neurons have many wonderful properties. Main of them is the high functionality and simplicity of learning. It will be shown that using simple single-layered MVN-based neural network it is possible to identify the type of the distorting operator. We will consider four types of blur: defocus, rectangular, motion and Gaussian ones. The preliminary results for the blur and type of blur identification problem have been presented in[6], but just motion and Gaussian blur have been considered, also as the restoration technique itself has not been presented.

After a type of blur and its parameters identification the image can be restored using several kinds of methods. Some fundamentals of image restoration will be considered. The image restoration (using the information obtained by the neural network) by Tikhonov regularization will be presented.

2. GENERAL APPROACH TO THE RESTORATION PROBLEM

The image restoration problem is usually formulated in the following way. The restoration of the image is reduced to the obtaining of the non-distorted image $z(\zeta,\eta)$ from the given equation:

$$Az + n = u(x,y) + n(x,y) = \tilde{u}(x,y),\ (x,y) \in \Omega, \tag{1}$$

where $A: Z \to U$ (Z, U –metric spaces) is a given linear or nonlinear operator, $z \in Z$, $u \in U$, $n(x,y)$ is a noise, $\tilde{u}(x,y)$ is an output distorted image.

The most universal principles of solving this problem are formulated in the statistical estimation theory, and in the theory of solving of the ill-posed problems[2, 7, 8]. Beside the existing general rules, the methods of restoration are based on the use of the specific features of a particular problem (simplicity of the distortion operator, existence of a known background, etc.). However, regardless of what approach we use, the restoration problem can not be solved using empirical methods, that are so effectively applied to other problems of the image processing. The restoration problem is a typical inverse problem of mathematical physics, and as any other inverse problem it can be solved only by the corresponding basic mathematical methods.

It is evident that whatever method we use to obtain the restored image, it must comply in a certain way with the basic equation (1). I.e., it must provide the closeness of the left and the right sides in (1). So the most general formulation of the restoration problem can be reduced to the following functional minimization:

$$z^* = \inf_{z \in Z} \rho_U(Az, \tilde{u}), \quad (2)$$

where ρ_U is a certain metric in *U*. In general it is possible to use different definitions of a distance ρ_U between two images.

It is easy to show that the solution of the optimization problem (2) is not unique even when operator *A* and the distorted image *u*(*x,y*) are given exactly, without any noise. We should use a priori information about the required $z(\zeta, \eta)$ to choose a unique and stable solution from the whole solutions set, as for any underdetermined problem.

The simplest way to guarantee the uniqueness and stability of the solution is to formulate "a priory" information about the original image using a functional $\Omega(z)$ that possesses stabilizing properties[6]. In this case the image restoration problem can be reduced to the conditional or unconditional optimization problem, in particular to the Tikhonov minimization:

$$z^* = \inf_{z \in Z} \{\rho_U(Az, \tilde{u}) + \alpha\Omega(z)\}, \quad (3)$$

where α is the parameter of the regularization. Usually it is assumed that the original image is a smooth function with respect to Sobolev space, and a stabilization functional in (3) is $\Omega(z) = \|z\|^q_{W^p_q}$. However, it is possible to obtain the new results of restoration considering an image set as a set of the functions of bounded variations[7]. From this definition it follows that:

$$z^* = \inf_{z \in Z} \{\rho_U(Az, \tilde{u}) + \alpha Var(z)\}$$

If a space *U* is defined as the Euclidian space with respect to the norm (*u*,*Bu*), where *B* is a positive defined operator, we obtain the following:

$$z^* = \inf_{z \in Z} \{\|Az - \tilde{u}\|^2_B + \alpha\Omega(z)\}. \quad (4)$$

It should be noted that the statistical methods used in image restoration lead to the optimization problems similar to (3). Thus using Bayes strategy or MAP test we obtain the optimal estimation in the following form:

$$z^* = \inf_{z \in Z} \{-\ln q(Az - \tilde{u}) - \ln p(z)\}, \quad (5)$$

where *p(z)* and $q(\xi)$ are a priori probability densities of an original image $z(\zeta, \eta)$ and the additive noise $\xi(x, y) = Az - \tilde{u}$.

The main essential difference between the regularization method of image restoration (3) and the statistical method (5) is the existence of the regularization parameter α in (3). It is necessary to point out that the opportunity of obtaining a family of solutions that depend on a parameter α is very important. This allows us to control the visual quality of the image restoration interactively in the absence of a mathematical criterion of visual image quality.

3. IMAGE RESTORATION TECHNIQUES

The most universal techniques of solving the restoration problem are given in the regularization theory[2, 8]. In this case the solution of restoration problem comes to finding out the unconditional extremum, in particular, to searching the minimum of the functional (3).

Unfortunately, it is problematic to solve the minimization problem in the general two-dimensional case using a direct implementation of the minimization. The possibility of simplification the minimization task is usually based on the usage of specific features of the integral operator A. The imaging system in signal processing is usually described by a homogeneous operator. If in this case the degraded image $u(x, y)$ is given on the whole of plane $(x, y) \in (-\infty, \infty)$ the equation (1) comes to the convolution type equation:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x-\xi, y-\eta) z(\xi,\eta) d\xi d\eta + n(x, y) = \tilde{u}(x, y), \quad -\infty < (x, y) < \infty . \tag{6}$$

The equation (6) can be solved by using the Fourier transform technique. As the matter of fact, the degraded image $u(x, y)$ is defined on a bounded domain W. It does not permit to apply the Fourier transform directly to the equation (6). To overcome this restriction an additional procedure is required to extend the definition of the degraded image onto the whole plane $(x, y) \in (-\infty, \infty)$.

The convolution nature of the equation (6) implies that its equivalent Fourier representation is

$$\tilde{U}(f_x, f_y) = H(f_x, f_y) Z(f_x, f_y) + N(f_x, f_y), \tag{7}$$

As it was mentioned above, an effective implementation of the general solution of the image restoration problem given by equation (3) is difficult due to very large image dimensions. This drawback significantly restricts the wide usage of Fourier approach. In particular in image restoration the Fourier technique still is the most dominant[9].

We would like to note that the Fourier technique has at least two drawbacks: (a) this method is only applicable for solving of equations of the convolution type and (b) in a direct way, it can only be used for implementation of linear restoration algorithms when ρ_U and $\Omega(z)$ in (3) are quadratic forms.

It is known[8] that the general linear solution of the equation (6) can be written as

$$\hat{z}(\xi,\eta) = \iint K(\xi - s, \eta - t)\tilde{u}(s,t) ds dt, \tag{8}$$

where the kernel of the inversion has the following form

$$K(s,t) = \frac{1}{4\pi^2}\iint R(f_x, f_y) e^{i(sf_x + tf_y)} df_x df_y = \frac{1}{4\pi^2}\iint \frac{H^*(f_x, f_y) e^{i(sf_x + tf_y)}}{\left|H(f_x, f_y)\right|^2 + \Psi(f_x, f_y)} df_x df_y, \tag{9}$$

and $\Psi(f_x, f_y)$ is a given function.

Thus, $z(\xi,\eta)$ can be found by taking the inverse Fourier transform of

$$Z(f_x, f_y) = R(f_x, f_y)\tilde{U}(f_x, f_y).$$

When the noise $N(f_x, f_y)$ in (7) is absent, the obvious filter is the inverse filter

$$R(f_x, f_y) = H^{-1}(f_x, f_y)$$

However, the inverse filter may not exist if $H(f_x, f_y)$ possesses singularity. In the presence of noise the optimal restoration filter known (in the MSE criteria) is the least squares filter or the Wiener filter

$$R(f_x, f_y) = \frac{H^*(f_x, f_y)}{\left|H(f_x, f_y)\right|^2 + \dfrac{S_{nn}(f_x, f_y)}{S_{zz}(f_x, f_y)}}$$

where $S_{nn}(f_x, f_y)$ and $S_{zz}(f_x, f_y)$ are the noise and the object power spectra, which are assumed to be known. It is also assumed that the noise added is a white noise, i.e., its spectral density is constant, and the picture and the noise are uncorrelated. This method works well only for images with a high signal to noise ratio (SNR), which is defined to be the ratio between the variance of the picture and the variance of the noise, and performs poorly for images with low SNRs.

In our experiments as an approximate solution to equation (6) (with approximate $\tilde{A}$ and $\tilde{u}(x, y)$) an extremal of the Tikhonov's linear functional (4) was taken, with

$$\Omega(z) = \iint [z^2 + c(\frac{\partial z}{\partial x})^2 + c(\frac{\partial z}{\partial y})^2]^2 dxdy \qquad (10)$$

In this case the restored image $\hat{z}(\xi, \eta)$ is defined by the formula (8) where the kernel of the inversion has the following form

$$K(s,t) = \frac{1}{4\pi^2}\iint R(f_x, f_y)e^{i(sf_x + tf_y)}df_x df_y = \frac{1}{4\pi^2}\iint \frac{H^*(f_x, f_y)e^{i(sf_x+tf_y)}}{\left|H(f_x, f_y)\right|^2 + \alpha(1+\beta(f_x^{\ 2} + f_y^{\ 2})}df_x df_y, \qquad (11)$$

As it is known[2] regularization parameter values $0 < \alpha < 1$ and $0 < \beta < 1$ depend on the noise level and on the targets of research. In restoration of the modeled distortions, when input data are given exactly, without errors, the restoration was started from the inverse filtering with $\alpha = 0$. When noise was present, the regularization parameters α and β in (11) were taken according to visual quality of the result, but not according to mathematical criteria[8].

4. MULTI-VALUED NEURON AND ITS LEARNING

As it was mentioned above, we will use a neural network based on multi-valued neurons (MVN) for the blur and blur parameters identification. Let us consider some fundamentals of MVN, its learning and networks based on it.

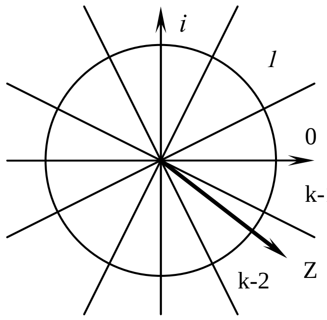
MVN has been introduced in[10] and it is deeply considered in[5]. MVN performs a mapping between n inputs and a single output. The mapping is described by multiple-valued (k-valued) function of n variables $f(x_1, ..., x_n)$ via their representation through n+1 complex-valued weights $w_0, w_1, ..., w_n$:

$$f(x_1, ..., x_n) = P(w_0 + w_1x_1 + ... + w_nx_n) \qquad (12)$$

where $x_1, ..., x_n$ are variables, on which the performed function depends. Values of the function and of the variables are k^{th} roots of unity: $\varepsilon^j = \exp(i2\pi j/k)$, $j \in \{0, k\text{-}1\}$, i is an imaginary unity. P is the activation function of the neuron:

$$P(z) = \exp(i2\pi j/k),\ if\ \ 2\pi j/k \le \arg(z) < 2\pi\ (j+1)/k \qquad (13)$$

where j=0, 1, ..., k-1 are values of the k-valued logic, $z = w_0 + w_1x_1 + \ldots + w_nx_n$ is the weighted sum , arg(z) is the argument of the complex number z. The equation (13) is illustrated in Fig. 2.



$$P(z) = \varepsilon^{k\text{-}2}$$

Fig. 2. Definition of the MVN activation function. If the weighted sum is equal to z then the output is equal to $\varepsilon^{k\text{-}2}$

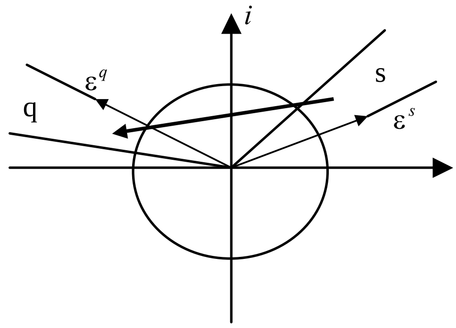


Fig. 3. The problem of MVN learning

For MVN, which performs a mapping described by the *k*-valued function, we have exactly *k* domains. Geometrically they are the sectors on the complex plane (Fig. 2).

The MVN learning is based on the same background as the perceptron learning. It means that id the weighted sum is going to the "incorrect" domain then the weights might be corrected in some way to direct the weighted sum into the correct domain. Let us consider this process in the details. It is illustrated in Fig. 3. If the desired output of MVN on some element from the learning set is equal to ε^q then the weighted sum should be exactly in the sector number *q*. But if the actual output is equal to ε^s, it means that the weighted sum is currently in the sector number *s*. A learning rule should correct the weights to move the weighted sum from the sector number *s* to the sector number *q*. The following correction rule for learning of the MVN has been proposed[5]:

$$W_{m+1} = W_m + C_m\,(\varepsilon^q \text{-} \varepsilon^s)\ \overline{X}\ , \tag{14}$$

where W_m and W_{m+1} are the current and the next weighting vectors, $\overline{X}$ is the complex-conjugated vector of the neuron's input signals, C_m is the scale coefficient.

Learning algorithm, which is based on the rule (14) is very quickly converging.

5. MVN-BASED NEURAL NETWORK AND ITS APPLICATION TO THE BLUR IDENTIFICATION

We will use here a single-layer MVN-based neural network, which contains the same number of neurons as a number of classes we have to classify[5, 11]. Each neuron has to recognize pattern belonging to its class and to reject any pattern from any other class. The architecture is presented in Fig. 4.

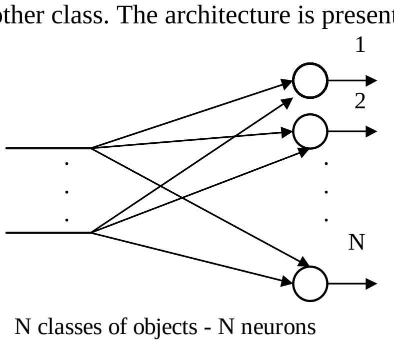
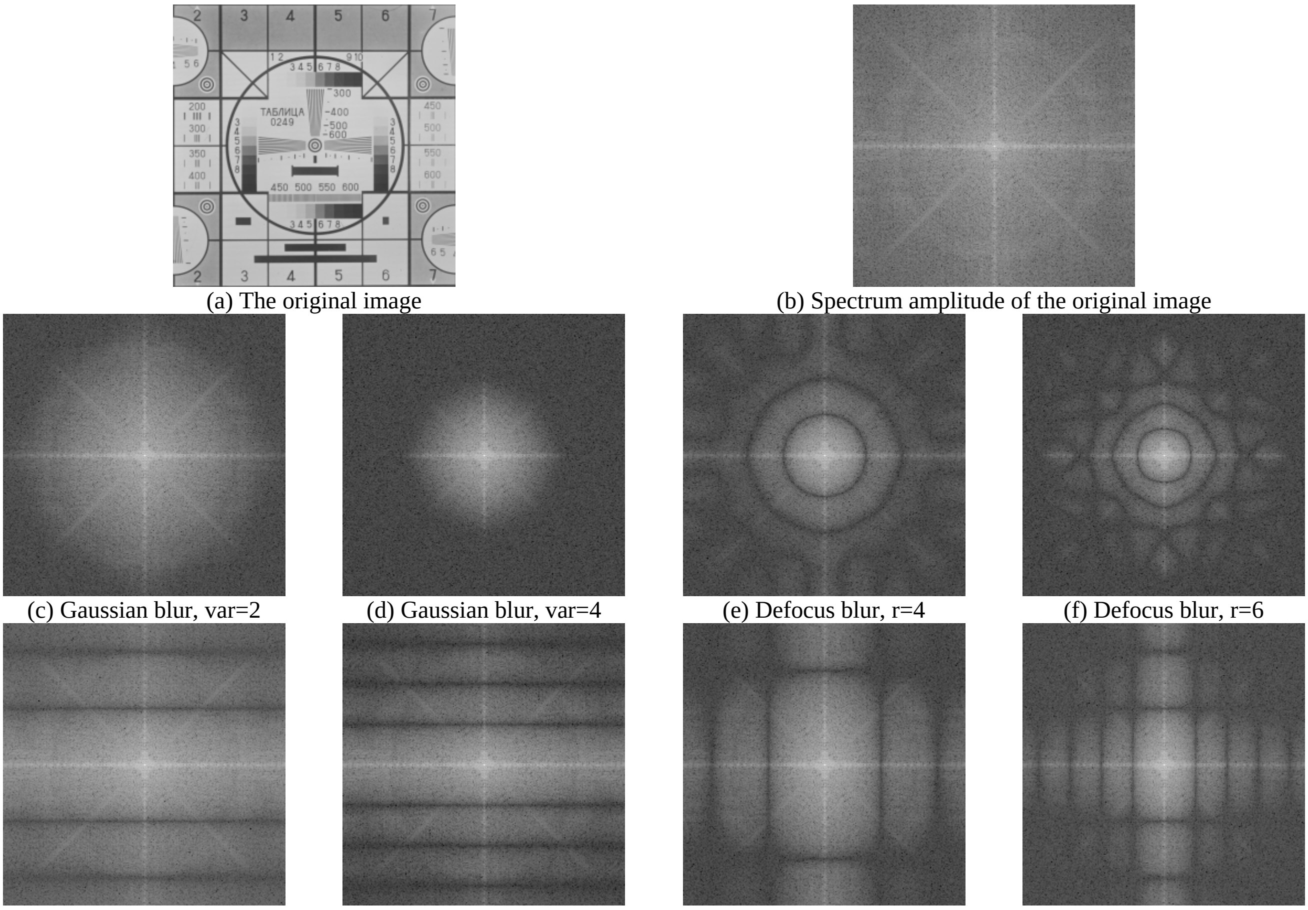


Fig. 4. MVN based neural network for pattern recognition

As it was mentioned above, the blur leads to the specific distortion of the image Fourier spectrum amplitude. This distortion can be clearly detected as a disappearance of the high frequency part of the image Fourier spectrum amplitude. A character of this disappearance is very specific for the different types of blur. Fig. 5 illustrates this important property.

Thus the Fourier spectrum amplitude contains the important information about the signal properties (existence of noise, blur, etc.). This means that it is possible to use it for the identification of blur, its type and parameters. The idea to analyze the spectrum amplitude using neural network is based on the following considerations. We do not need to analyze data itself, data as a formal set of numbers. Our target is to "catch" the behavior of the spectrum amplitude. A neural network has to be taught to distinguish a specific behavior corresponding to each type of blur independently on the particular image. Since we do not know a low, by which this behavior might be described in the analytical form, the use of the neural network is almost ideal solution. Indeed, a fundamental property of the neural network is its ability to accumulate the knowledge about the objects using the learning process, in the conditions, when a mapping between its inputs and outputs can not be defined in analytical form.



(a) The original image (b) Spectrum amplitude of the original image

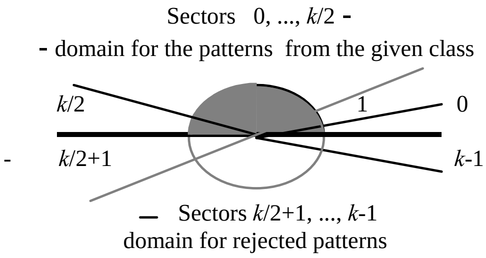
(c) Gaussian blur, var=2 (d) Gaussian blur, var=4 (e) Defocus blur, r=4 (f) Defocus blur, r=6

(g) 1D Motion blur, 4 pixels (h) 1D Motion blur, 6 pixels (i) Rectangular blur 4x2 (j) Rectangular blur 8x4

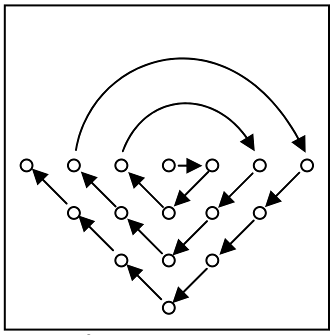
Fig. 5. Fourier spectrum amplitude and its distortion implied by the different blurs

To organize the learning process for the network presented in Fig.4, the following reservations of the domains has been used. The output values 0,..., *k/2*-1 of the i^{th} neuron correspond to the classification of object as belonging to i^{th} class. The output values *l*,..., *k*-1 correspond to the classification of object as rejected for the given neuron and class (*k* is taken from (13)). This reservation of the domains is shown in Fig.6a.To prepare the data for the learning and further recognition we used the normalization procedure based on the logarithmic quantization.

To ensure the accumulation by the neural network a stable knowledge about the blur, those spectrum amplitudes have to be used, which belong either to the regions, where the amplitude is not distorted and where it is distorted. The best way to do it is the extraction of the amplitude values using the “zigzag” rule (Fig. 6b). It is necessary to take into account that the 2D Fourier spectrum amplitude of the image (which is a real-valued signal) has a central symmetry property. It means that we need to use just a half of the amplitudes corresponding to each frequency (Fig. 6b).



(a) Reservation of the domains for the recognition



(b) Extraction of the Fourier spectrum amplitudes using the "zigzag" rule according to their frequency ordering

Fig. 6. Reservation of the domains and extraction of the data for the blur identification

To test abilities of the blur and blur parameters identification, we used four kinds of blur: Gaussian, Defocus, 1D horizontal motion and rectangular ones. The images of different nature have been used: landscapes, satellite optical images, face images. The images were blurred by the mentioned blurs with the different parameters. To make our model more realistic, we corrupted the images by zero-mean Gaussian noise with the dispersion 0.3σ.

For the blur identification 140 images have been taken. Since a spectrum amplitude behavior for the rectangular and defocus blurs is often very similar, we use two-stage blur identification. Three classes have been considered on the first stage: Gaussian, 1D motion, Rectangular/Defocus. The testing results are very good: 95-97% of the correct identification. If the corresponding blur was classified as "Rectangular/Defocus", we used additional single-layered neural network from two neurons (with the same architecture, see Fig.4) to identify exactly is it rectangular or defocus. The results are good. For the 94% of the images the identification is correct. The rest of 6% corresponds to defocus with a small radius (1) and the rectangular blur 1x1.

For blur parameters recognition 25 classes with different rectangular blurs were created. Steps for rectangular blur parameters are equal to 2 pixels in both directions (i.e. 1x1, 1x3, …, 1x9, 3x1, …, 3x9, …. 9x9). These 25 classes contain 500 images in the testing set (20 per class), and 12 images per class in the learning set, which is 300 images. To test the Gaussian blur variance identification 5 classes were distinguished (with variance 1 to 5, 200 images per class). Respectively, the defocus blur radius identification has been tested for the 15 classes (radius from 2 to 16, 500 images per class), 1D horizontal motion blur has been tested for the 16 classes (motion from 3 up to 18 pixels) and rectangular blur parameters identification has been tested for the 32 classes (1x1, 1x2, 2x1, etc).

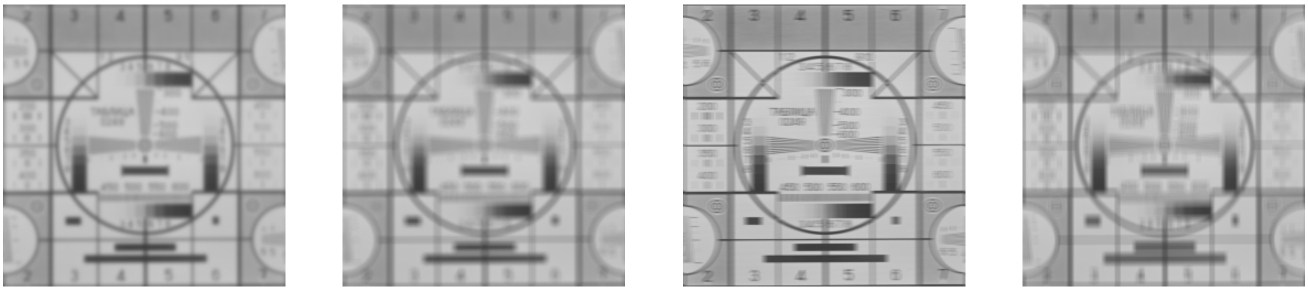
The results of the testing are summarized in the Table 1. It is evident that they are very promising.

Table1. Blur parameters identification testing.

Type of blur	Average level of successful parameters identification
Gaussian blur	93.5 %
Defocus blur	94.1 %
Motion blur	98.1%
Rectangular blur	95.6%

6. SIMULATION RESULTS

The importance of the proposed solution for the blur identification is very high. The image restoration problem cannot be solved without the effective solution of blur and blur parameters identification problem. Indeed, it is impossible to identify the blur, moreover, its parameters visually. The blurred images often are very similar to each other from visual point of view (see Fig. 7)

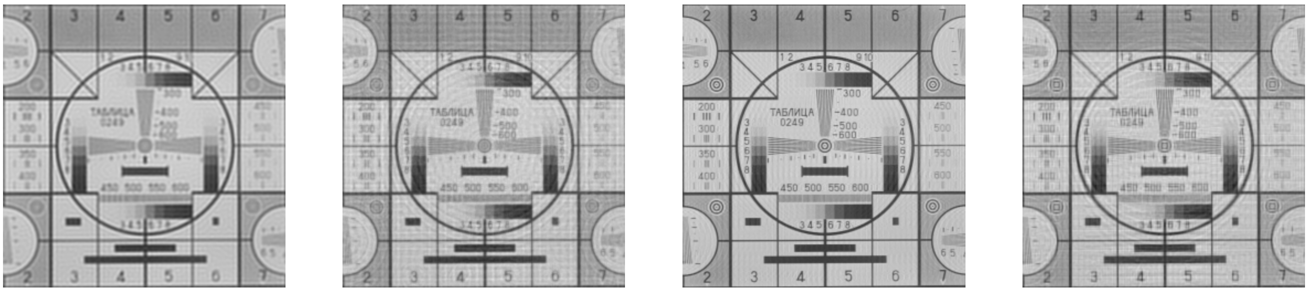


Gaussian blur, var=4 | Defocus blur, radius=4 | 1D motion blur, 4 pixels | Rectangular blur 8x4

Fig. 7. The examples of the different blurs for the original image presented in Fig. 5a

So to identify a type of blur and then to identify its parameters we used the neural network and the algorithm presented here. Then the images have been restored using the Tikhonov's regularization (8)-(11).

Let us consider two examples of the image restoration. The first one is presented in Fig. 8. This is the restoration of the artificially blurred images from the Fig.7. These images didn't participate in the learning process. The blur and its parameters were correctly identified in each case using the neural network. One may compare the restoration results to the original image in Fig 5a and to the corresponding blurred images in Fig. 7.



Restoration Gaussian blur, var=4 | Restoration Defocus blur, radius=4 | Restoration 1D motion blur, 4 pixels | Restoration Rectangular blur 8x4

Fig. 8. Restoration of the artificially blurred images presented in Fig. 7 using the blur identification and the Tikhonov filter

(a) The original blurred image

(b) The restoration result. The blur has been identified as rectangular one, 3x1.
Tikhonov filter has been used for the restoration

Fig. 9. Restoration of the originally (optically) blurred image

The next example shows the restoration of the originally (optically) blurred image. This example is shown in Fig. 9.

The same technique is applicable to the color images. The blur and blur parameters identification can be applied separately for the color channels. The restoration can also be performed separately for each channel.

A special case is the processing of the images with the dimensions not equal to power of 2. It is a usual case in real life, but a special case from the point of view of implementation because of well known fact that the fast Fourier transform algorithms exist just for the dimensions equal to power of 2. A usual solution in this case is image extension to the closest power of 2 in both directions. At the same time a simple even extension of the image can't be recognized as an optimal solution because it leads to the appearance of the boundary effects on the restored image. To avoid these effects we suggest using the original apodization technique.

It is a well-known fact, that any correction of the spectrum is immediately followed by appearance of edge effect on the processed image. To handle this effect some methods exist, for example symmetric extension or periodic extension of the image. But both of these algorithms do not remove sharp discontinuities on the image.

The Fourier transform itself assumes that signal is periodic and thus if left side of the image doesn't join right side smoothly (same for the top and bottom), in case of big edge difference, edge effect will appear. The proposed algorithm eliminates this edge difference.

For one-dimensional case idea is following. Suppose we have function $f(x)$ given on the interval $[a,b]$. Then $f(a+b-x)$ will be an inverse of $f(x)$ on the same interval. Let's build simple symmetric extension of $f(x)$ to the left and to the right of $[a,b]$:

$$g(x) = \begin{cases} f(x), & x \in [a,b] \\ f(a+b-[x+\delta]), & x \in [a-\delta, a) \\ f(a+b-[x-\delta]), & x \in (b, b+\delta] \\ 0, & otherwise \end{cases} \qquad (15)$$

$$\delta = b - a$$

Now let us build the function that will smooth the edges:

$$T(x) = \begin{cases} 1, & x \in [a,b] \\ \cos\frac{\pi(x-a)}{\delta}, & x \in [a-\delta, a) \\ \cos\frac{\pi(x-b)}{\delta}, & x \in (b, b+\delta] \\ 0, & otherwise \end{cases} \qquad (16)$$

Let us smooth the edges:

$h(x)=g(x)T(x)$ (17)

And now let's make a periodical function $F_{per}(x)$ with period 2δ:

$$h_k(x) = h(x - 2k\delta)$$
$$F_{per}(x) = \sum_k h_k(x) \qquad (18)$$

Because in image processing we are dealing with the functions with finite support, the following function will be subdued to the Fourier transform:

$$F(x) = \begin{cases} F_{per}(x), & x \in \left[a - \delta/2, b + \delta/2\right] \\ 0, & \text{otherwise} \end{cases} \qquad (19)$$

The idea of apodization for two-dimensional functions is the same, and can be formulated as two 1D apodizations: horizontal and vertical. This apodization is preferable when image size is not a power of two (which is the limitation of FFT), and also can be used to extend image size to the next power of two if it is needed. Fig. 10 shows the example of this apodization applied to an image of the dimensions 200x200 to extend to 256x256. Just for comparison Fig. 10(b) contains image extended symmetrically in all directions. Both images (b) and (c) are shifted, so it is visible how the borders will join. Note the sharp discontinuities on the image (b) and how they are smoothed on image (c).

(a) Original image (200x200)

(b) Symmetric extension of original (256x256) shifted by 128 pixels in every direction

(c) Described Apodization (256x256) shifted by 128 pixels in every direction

Fig. 10. Apodization Demonstration

7. CONCLUSIONS AND FUTURE WORK

The main result of the presented work is the effective and original image restoration technique carried out in the paper. The key point of this technique is blur (distorting operator) and its parameters identification using the neural network based on multi-valued neurons. The results of this identification are used as the main parameters for the image restoration using the Wiener, Tikhnov or inverse filters. The future work will be directed to the consideration of more blurs, including the combinations of several different blurs and to the development of the restoration technique.

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